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Research Article

A Dynamic Analysis of Convective Weather Using Generalized Dynamic Equations and the Scale Transformation Mechanism Related to Condensation and Freezing Processes

Xingrong Wang[1]*, Yushu Zhou[2], Yong Huang[1] and Zhe Zhang[2]

[1]Anhui Meteorology Institute, Anhui Key Lab of Atmospheric Science and Satellite Remote Sensing, Hefei, Anhui, China

[2]Key Laboratory of cloud-precipitation physics and severe storms (LACS), Institute of Atmospheric Physics(IAP), Chinese Academy of Sciences(CAS), Beijing, China

Abstract

To figure out the influence of non-conserved Generalized Potential Vorticity (GPV) caused by latent heating and dry diabatic heating on convective weather, a GPV equation is studied. By comparing latent heating and dry diabatic heating term with nonlinear advection term, three kinds of processes are discussed, namely adaptation, evolution, and excitation (elimination), associated with the conservative, quasi-conservative, and non-conservative GPV, respectively.

The analysis points out that:

1. Air movement transforms from smaller to larger scale by very fast adaptation process with the addition of frictional effect when the order of latent heating A and dry diabatic heating term B is lower than Rossby number R [$O(f^{-1}(A+B)/P_0)<O(R)=O(W/FH)$].
2. Air movement transforms from larger to smaller scale by very fast excitation process or from smaller to larger scale by very fast elimination process when the order of A+B is higher than R [$O(f^{-1}(A+B)/P_0)>\max(O(R),O(W/FH))$].
3. Either of both kinds of scale transformation process will constantly go along till air movement is back to evolution process.

For each transformation process of convective weather, its dynamic causes, conditions and main features are theoretically analyzed and described, many puzzles about convective cloud may be understood and explained.

Keywords: Cumulonimbus; Cumulus; Generalized dynamic equations related to condensation and freezing processes; Scale transformation mechanism

***Corresponding author:** Xingrong Wang, Anhui Meteorology Institute, Anhui Key Lab of Atmospheric Science and Satellite Remote Sensing, Hefei, Anhui, China, E-mail: wangxr_ahqks@yahoo.com

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Introduction

Each year, many people are killed or seriously injured by severe meso-microscale convective weather e.g., thunderstorm. So many studies have contributed to their forecasting and understanding, for example, [1] reviewed the current state of knowledge on surface-forced convection initiation and then describe some of the outstanding issues in convection initiation that partially motivated IHO_2002 [2], revealed that the boundary layer vertical velocity field was dominated by thermals rather than by circulations associated with the mesoscale boundaries. Weckwerth and Parsons DB [3], pointed that Boundary layer circulations are sometimes precursors to deep convective development Doswell [4] presented an overview of the history of research related to severe convective storms with a particular emphasis on the connection between this research and forecasting. Gao, et al., [5] found that the cold, dry air with high potential vorticity shows up prior to the start of precipitation and persists and extends downward to lower levels during the raining periods. Wakimoto and Murphey [6] presented an analysis of a dryline that did not initiate convection during the observational period ; Wakimoto and Murphey [7] presented an analysis of six convergence boundaries observed during the International H(2)O Project (IHOP_2002) and suggested that it is not necessary for two boundaries to collide in order for thunderstorms to develop. Solenoidally generated horizontal circulations can produce conditions favorable for convection initiation even if the boundaries remain separate.; Paul Markowski and Yvette Richardson [8], which the contains all of what most of us would consider core midlatitude mesoscale meteorology. Luo YL, et al., [9] studied “The initiation and organization of a quasi-linear extreme-rain-producing Mesoscale Convective System (MCS) along a mei-yu front in east China during the midnight-to-morning hours of 8 July 2007” ; Yu, et al., [10] introduced the advances in the nowcasting techniques on thunderstorms and severe convection; Wang H, et al., [11] introduced Initiation, maintenance, and properties of convection in an extreme rainfall event during scmrex. These studies have made significant progress in many areas and have contributed to the forecasting and understanding of severe convective weather. However, due to its serious harm to humans and its current unpredictability, these advances are still insufficient.

According to the traditional atmospheric dynamical equations, from Ertel H and Wu, et al., [12,13] if excluding the diabatic heating and friction, whether moist potential vorticity in relative humidity r=1 (MPV) or dry potential vorticity in r<1 (DPV), both are conservative because the solenoidal term of MPV.

$$A_{MPV}=\left\{\frac{R_d}{p}\left(\frac{p}{p_0}\right)^{-R_d/C_p}\right\}\cdot(\nabla p\times\nabla\theta)\cdot\nabla\theta\exp\left(\frac{L_d q_s}{C_p T}\right)=0 \tag{1}$$

and the solenoidal term of DPV

$$A_{DPV}=\left\{\frac{R_d}{p}\left(\frac{p}{p_0}\right)^{-R_d/C_p}\right\}\cdot(\nabla p\times\nabla\theta)\cdot\nabla\theta=0 \tag{2}$$

Here C_p is the specific heat at a pressure p, R_d, θ, T, q_s and L_d are specific gas constant for dry air, potential temperature, temperature and saturated specific humidity and the specific latent heat of vaporization, respectively.

It means that all current popular research on convective weather is constrained by the conservation of potential vortices. From Wang and Huang [14], the Generalized Potential Vorticity (GPV) is found non-conservative in some very thin transitional areas. So, there are potentially useful applications of non-conservative property of the GPV in the convective storm realm that are as yet unrealized.

According to Doswell CA [15], despite the fact that there might be a layer in the atmosphere that has positive values of Convective Available Potential Energy (CAPE), if the parcel does not reach or begin rising to that level, the most significant convection that occurs in the Free Convective Layer (FCL) will not be realized. This can occur for numerous reasons. Primarily, it is the result of a cap, or convective inhibition (CIN/CINH). Processes that can erode this inhibition are heating and forcing of the Earth's surface. Such forcing mechanisms encourage upward vertical velocity, characterized by a speed that is relatively low to what you find in a thunderstorm updraft. Because of this, it is not the actual air being pushed to its FCL that 'breaks through' the inhibition. So it can be determined that the heating and forcing of the Earth's surface is a toggle condition, and is a necessary but insufficient condition to break CIN. In fact, discontinuity in the latent heat term is generally found upon considering the transitional areas between unsaturated and saturated air as well as air below and above the freezing point. Traditionally, discontinuity in the latent heat term is represented by the Heaviside step function ($\delta(0,1)$) and increases the difficulty in ordinary theoretical analyses. Certain discussions are constrained by saturated air and equilibrium airflow conditions, for example, Wu, et al., (1995), Ooyama and Schubert, et al., [13,16,17].

Wang and Wu [18] introduced the condensation probability function r^k (r is relative humidity) based upon the observational evidence presented by Mason [19] to represent the continuity of the variations observed in the discontinuity of the condensation latent heat term. Given that the condensation process may occur at r = 78%, Wang XR, Wang ZX, and Shi CE [20,21] observed r^k is close to zero in r < 78% and r^k= 1 in r = 1 when $k \approx 9$. The condensation in air is well represented. Following these discussions, the dynamic equations of nonuniform saturated moist air were provided.

From the discuss of tendency equation of nonuniform saturated moist potential vorticity or Generalized Moist Potential Vorticity (GMPV), in some very thin transitional areas (sub-saturated area ($0.78 < r < 1$), GMPV is found non-conserved because of the introduction of the condensation probability function even in frictionless and moist adiabatic air motion [22], Based on this conclusion, the scale transformation mechanism of nonuniform saturated moist air [23] is discussed.

On the scale transformation mechanism, an examination of the non-conservation conditions of the nonuniform saturated moist potential vorticity theoretically proved four necessary occurrence conditions for sudden heavy rain (thunderstorm), which are confirmed with data [24]. Their analyses showed that the decline of the cold moist air stream above CIN can erode CIN and make $\partial\theta/\partial z$ being fast and sufficiently close to zero. It makes convective available depth (satisfying sufficient and necessary conditions of the moist excitation process) extending beyond the CIN top before it was destroyed after the CIN has eroded, once this depth extends to a deep convective depth, the thunderstorm occurs, which means that the thicker the depth of the eroded CIN is, the stronger the excited convective positive vorticity system. In fact, before the occurrence of essentially all the thunderstorm, except the heating and forcing of the Earth's surface, the cold moist air stream above the CIN has sunk, which means that it is possible to predict the time and intensity of severe convective weather based on these symptoms [24,25].

According to the above research results, it can be determined that a toggle process [the heating and forcing of the Earth's surface pushing the parcel to some very thin subsaturated area ($1 > r > 0.78$)] with the eroded process of CIN [the decline of the cold moist air stream above CIN] is an insufficient and necessary condition to break CIN. This also shows that the restrictions of the conservation law of the potential vortices must be broken down to make greater progress in convective weather forecasting and understanding.

The theory and application of nonuniform saturated moist air dynamics and the corresponding scale transformation mechanism on the occurrence conditions of thunderstorm has recently seen great progress. However, the difficulties in the discontinuity of the freezing latent heat term still limits the theoretical understanding and interpretation of the effect of freezing processes on convection weather.

Regarding the discontinuity of the freezing latent heat term, besides the condensation probability function [18,22], the freezing probability function is introduced into the dynamic framework based on the statistical thermodynamic fluctuation theory to derive the generalized thermodynamic equation in relation to the condensation and freezing processes [14]. "From the discussion of tendency equation of the GPV, in some very thin transitional areas [sub-saturated area ($0.78 < r < 1$ and $T > 277.13K$), subfreezing saturated area (269.13K < T < 277.13K and r = 1) and the subfreezing and subsaturated area (269.13K < T < 277.13K and $0.78 < r < 1$)], GPV is found non-conserved because of the introduction of the condensation probability function and freezing probability function, even in frictionless and moist adiabatic air motion"[15].

To figure out the influence of non-conserved GPV caused by latent heating and dry diabatic heating on convective weather, the scale transformation mechanism related to condensation and freezing processes is discussed in Section 2. Followed by the discussion mentioned above, a dynamic analysis of convective weather is provided in Section 3. The conclusions are given in Section 4.

The Scale Transformation Mechanism Related to Condensation and Freezing Processes

It is necessary to point out that the scale transformation mechanism proposed by Wang and Wei [23] has certain scientific significance, but there are two main problems because of the limitations of the conditions at the time.

1. It only involves the condensation process without including the freezing processes because applying to nonuniform saturated moist air.
2. The elimination process is not considered when discussing non-uniform saturated moist PV non-conservation process.

For this reason, the scale transformation mechanism will be revisited.

If generalized potential temperature θ^* is defined as follows:

$$\theta^* = \theta \exp\left(\frac{(L_i\varphi(T) + L_d)}{C_p T} r^k q_s\right) \quad (3)$$

Where L is the specific latent heat, which is expressed as L_d and the specific latent heat of fusion L_i, in certain cases, respectively; $\varphi(T)$ is the freezing probability function [14], which can be written as follows:

$$\phi(T) = \phi(273.15, T, 2.4) = \frac{1}{\sqrt{2\pi}2.4}\int_{-\infty}^{273.15} e^{\frac{-(T_{ax}-T)^2}{2(2.4)^2}} dT_{ax} \quad (4)$$

According to Wang and Huang [14], the Generalized Potential Vorticity (GPV) tendency equation can be derived and expressed as follows:

$$\frac{dP_m}{dt} = \alpha(\nabla p \times \nabla\alpha)\cdot\nabla\theta^* + \alpha\vec{\varsigma}_a \cdot \nabla Q \quad (5)$$

where

$$P_m = \alpha\overrightarrow{\varsigma_a}\cdot\nabla\theta^* \quad (6)$$

Equation (5) also has the following form

$$\frac{dP_m}{dt} = P_m(A+B)/P_0 \quad (7)$$

Where

$$A = \left\{\frac{R_d}{p}\left(\frac{p}{p_0}\right)^{-R_d/C_p}\right\}\cdot(\nabla p \times \nabla\theta)\cdot\nabla\theta^* \quad (8)$$

$$B = \vec{\zeta}_a \cdot \nabla Q \quad (9)$$

$$P_0 = \vec{\zeta}_a \cdot \nabla\theta^* \quad (10)$$

here is GPV, α is the specific volume, $\vec{\varsigma}_a = \nabla\times\vec{V} + 2\vec{\Omega}$ is absolute vorticity vector, A is a solenoidal term that is related to the baroclinicity $(\nabla p \times \nabla\theta)$ and the gradient vector of the generalized potential temperature $\nabla\theta^*$, B is a dry diabatic heating term depending distribution vector of heating or cooling field ∇Q that excludes latent heating, and P_0 is the result of multiplying the vorticity vector with the gradient vector of the generalized potential temperature.

According to Wang and Huang [14], the GPV is found non-conservative in some very thin transitional areas because of the introduction of the condensation probability function and freezing probability function, even in frictionless and moist adiabatic air motion.

For figuring out the influence of non-conservative GPV caused by latent heating and dry diabatic heating on convective weather, according to the relativity principle of conservation, following Wang and Wei [12], using processing method similar to scale analysis, equation (7) can be expressed non-dimensionally as follows:

$$\varepsilon\frac{\partial P}{\partial t_1} + R\left[u_1\frac{\partial P}{\partial x_1} + v_1\frac{\partial P}{\partial y_1}\right] + \frac{W}{fH}\left(w_1\frac{\partial P}{\partial z}\right) = P\left(f^{-1}(A+B)/P_0\right) \quad (11)$$

Here Kibil number (ε) defined as $\varepsilon = 1/ft_0$, Rossby number R defined as $R = U/fL$. ε, R and W/fH are dimensionless numbers used in describing fluid flow and equal order in a single stable weather scale, but R and W/fH are not necessarily equal order as time tendency term because of the action of latent heating and dry diabatic heating when the order of A+B is higher than R $[O(f^{-1}(A+B)/P_0) > \max(O(R), O(W/fH))]$ (where t_0, U, L, W and H are, respectively, characteristic time, velocity, length vertical speed and vertical height scales of the phenomenon). "A small Rossby number signifies a system which is strongly affected by Coriolis forces, and a large Rossby number signifies a system in which inertial and centrifugal forces dominate. For example, in tornadoes, the Rossby number is large (≈ 10^3), in low-pressure systems it is low (≈ 0.1 – 1) and in oceanic systems it is of the order of unity, but depending on the phenomena can range over several orders of magnitude (≈ 10^{-2} – 10^2)"[26]; $u_1 = u/U$; v_1/U; $w = w/W$.

To distinguish whether there is a scale transformation process, with the aid of scale analysis, by comparing latent heating and dry diabatic heating term (A+B) with nonlinear advection term, following criteria can be used:

$$O(\varepsilon) = O(R) = O\left(\frac{W}{fH}\right) > O\left(f^{-1}(A+B)/P_0\right) \quad (12)$$

GPV, as a whole, is conserved;

When

$$O(\varepsilon) = O(R) = O\left(\frac{W}{fH}\right) = O\left(f^{-1}(A+B)/P_0\right) \quad (13)$$

Different from (12), (f^1 $(A+B / P)$ can be ignored, and it is not negligible in (13). So GPV, as a whole, is quasi-conserved; and when GPV, as a whole, is not conserved.

$$O(\varepsilon) = O\left(f^{-1}(A+B)/P_0\right) > \max\left(O(R), O\left(\frac{W}{fH}\right)\right) \quad (14)$$

It should be pointed out that, in the case where (14) is satisfied, that is, in non-conserved GPV processes, there are two cases, $A+B > 0$ and $A+B < 0$. A has two cases: A > 0 when the gradient has a component counter to the direction of the Baroclinic vector, and A < 0 when the gradient has a component in the direction of the baroclinic vector. B also has two cases depending on distribution vector of heating or cooling field ∇Q: $B > 0$ when $\nabla Q > 0$ and $B < 0$ when $\nabla Q < 0$. Therefore non-conserved GPV processes are named excitation or elimination processes.

The adaptation process

From equation (12), the sufficient and necessary conditions of the adaptation process is presented as follows:

$$O\left(f^{-1}(A+B)/P_0\right) < O(R) = O\left(\frac{W}{fH}\right) \quad (15)$$

Equation (15) indicates that local or small impacts caused by A+B can be omitted in the adaptation process. The majority of observed facts and dimensional analyses relate the spatial scale of the air motion to the destructive extent of equilibrium in the air pressure gradient force, Coriolis force, inertial force, and gravity. According to the aforementioned Lakshmi H. Kantha & Carol Anne Clayson [26], it can be found that smaller scales produce larger destructive extents. Therefore, in the adaptation process, any movement with a spatial $O(R) = O(W/fH) = (10^n)$ scale of can be deemed a perturbation on the larger scale of $O(R) = O(W/fH) = (10^{n-1})$.

Discussions on this topic are quite mature. From Yeh and Li [27], this process is a rapid adaptation process wherein the non-equilibrium of air pressure gradient force, Coriolis force, inertial force, and gravity disappears due to the dispersion of gravity and the sound waves. This adaptation process, with the addition of frictional effect, transforms a small-scale movement into a larger scale movement, and continues until equation (15) is no longer satisfied. In reality, small to large scale transformation proceeds very rapidly, which is a rapid process. For this reason, slow nonlinear advection impacts can also be omitted in this process.

Given that (A+B)/P0 can be omitted from equation (15), this process proceeds in a closed or quasi-closed system. The system entropy increases due to the dispersion of the gravity and sound waves according to the second law of thermodynamics. Therefore, the entire system is nondissipative.

The excitation (elimination) process

From equation (14), the sufficient and necessary condition of excitation (elimination) process is presented as follows:

$$O\left(f^{-1}(A+B)/P_0\right) > \max\left(O(R), O\left(\frac{W}{fH}\right)\right) \tag{16}$$

Equation (16) indicates that impacts of the nonlinear advection and the adaptation mechanism can be omitted in the excitation or elimination process. In actual air motion, the size of A and B does not affect the fulfillment of equation (16) so long as the character value of P_0 [$O(P_0)$] is small enough to satisfy equation (16). For the sake of discussion, (16) is rewritten as follows:

$$O(P_0) < O(f^{-1}(A+B)/\max\left(O(R), O(W/fH)\right) \tag{17}$$

The excitation or elimination process will occur if equations (16) or (17) are satisfied. Namely the perturbation caused by (A+B) can excite or eliminate a weather system as P0 approaches zero.

From equation (14), the temporal scale of excitation or elimination process mostly depends on the value of P0 and (A+B), which is much smaller than that of common air movement at the same spatial scale. This is also a rapid process.

When an excitation or elimination process occurs, GPV tendency equation (5) or (7) can be simplified using equation (16) as follows:

$$\frac{\partial P_m}{\partial t} = \alpha(\nabla p \times \nabla \alpha)\cdot\nabla\theta^* + \alpha\overrightarrow{\varsigma_a}\cdot\nabla Q = P_m(A+B)/P_0 \tag{18}$$

Using the expression of in equation (5) and in equation (10), equation (18) becomes:

$$\frac{P_0}{\alpha}\frac{\partial \alpha}{\partial t} + \frac{\partial \bar{\zeta}_a}{\partial t}\cdot\nabla\theta^* + \bar{\zeta}_a\cdot\frac{\partial \nabla\theta^*}{\partial t} = A+B \tag{19}$$

Generally, in actual air movement, $O(\Delta\alpha/\alpha) < 10^0$. Moreover, the sufficient and necessary condition of the excitation or elimination process states that $O(P_0)$ must be small enough ($\nabla P_0 \cong 0$) to proceed. Therefore, equation (19) can be simplified as follows:

$$\frac{\partial P_0}{\partial t} = \frac{\partial \bar{\zeta}_a}{\partial t}\cdot\nabla\theta^* + \bar{\zeta}_a\cdot\frac{\partial \nabla\theta^*}{\partial t} = A+B \tag{20}$$

Because A+B may either increase or decrease GPV, the equation (20) presents that the excitation process suddenly enlarges P_0 based on the increase in GPV and not due to the translations observed upstream when $A+B>0$. In this process, the ($A+B$) term changes the vorticity field and $\nabla\theta^*$ then changes P_0. As a result, the perturbation is induced with a smaller scale of $\max[O(R), O(W/fH)] = (10^{n+m})$ than the origin scale of $\max[O(R), O(W/fH)] = (10^n)$, the equation (17) also presents that the elimination process suddenly minimizes P_0 based on the decrease in GPV and not due to the translations observed upstream when A+B<0. In this process, the ($A+B$) term changes the vorticity field and $\nabla\theta^*$ then changes P_0. As a result, the perturbation is induced with a larger scale of $\max[O(R), O(W/fH)] = (10^{n-m})$ than the origin scale of $\max[O(R), O(W/fH)]$ (10). Either of both kinds of process will continue until equation (16) is no longer satisfied but rather the following relation must be satisfied:

$$O\left(f^{-1}(A+B)/P_0\right) \le \max\left(O(R'), O\left(\frac{W'}{fH'}\right)\right) \tag{21}$$

We define both kinds of process as an excitation or elimination process. Furthermore, the solenoidal term, A, is non-zero only in the sub-saturated area, subfreezing saturated area, and subfreezing and subsaturated area. In the subsaturated area, the excitation or elimination process caused by A is only related to the release of the latent heat of condensation, thereby deeming it the moist excitation or elimination process. In the subfreezing saturated area or in the subfreezing and subsaturated area, the excitation or elimination process caused by A is related to the release of freezing latent heat, and is called the ice excitation or elimination process. Conversely, the dry diabatic heating term, B, is only related to the diabatic heating without latent heating. Therefore, the excitation or elimination process caused by B is called the dry excitation or elimination process.

When the perturbation caused by ($A+B$) eliminates a convective available weather system, in which , even though the effects of elimination and adaptation processes are the same, and both are the transform process from a smaller to a larger scale, the mechanisms are different. The elimination process is achieved by the forcing of A or B. On the contrary, the adaptation process is implemented by the dispersion of gravity and the sound waves.

The omission of the in equation (14) suggests this process to be a linear process rather than a nonlinear one, and the system entropy decreases due to the excitation action of ($A+B$) or the system entropy increases due to the elimination action of ($A+B$). Therefore, it does not agree with the second law of thermodynamics. This process is in an open system, however, it is not in a thermodynamically steady state. Therefore, the entire system does not have a dissipative structure.

The evolution process

From equation (13), the sufficient and necessary condition of the evolution process is stated as follows:

$$O\left(f^{-1}(A+B)/P_0\right) = O(R) = O\left(\frac{W}{fH}\right) \tag{22}$$

Equation (22) indicates that there are three processes that impact in the evolution process, namely nonlinear advection, adaptation and excitation (elimination), which cannot be omitted. The mutual offset of the adaptation and excitation (elimination) impacts create an overall quasi-conservative GPV. Therefore, nonlinear advection seems plays an important role but is actually influenced by the adaptation and excitation (elimination).

Equation (13) shows interrelation and dependence on $(A+B)/P_0$ between the spatial and temporal scales, which suggest that $(A+B)$, the spatial scale, and the temporal scale do not change or change slowly. Therefore, this is a slow process and is not the scale transformation process.

Equation (13) also indicates that this process is not only in an open system but is also in a thermodynamically steady state because both $O(R) = O(W/fH)$ and $(A+B)$ cannot be omitted. Therefore, the system entropy does not change or changes slowly due to the mutual offset of the adaptation and excitation (elimination) impacts in addition to the frictional effect, thereby presenting the entire system with a dissipative structure.

In the evolution process, (A+B), GPV, the spatial scale, and the temporal scale do not change or change slowly. However, the change in $(A+B)$ and the nonlinear advection may make the sufficient and necessary condition of the adaptation and excitation or elimination process satisfied. If so, the corresponding adaptation and excitation or elimination process can occur.

A Dynamic Analysis of Convective Weather

According to the discussion above, before the excitation process of convective weather system, air movement is in a thermodynamically steady state because of CIN, from equation (21), there should be $(A+B) \cong 0$ and $P_0 = \vec{\zeta}_a \cdot \nabla\theta^* > 0$). If we exclude the dry diabatic heating in the very thin boundary of the cloud top and the fierce electric current dry diabatic heating related to lightning, $B = \vec{\zeta_a} \cdot \nabla Q \neq 0$ will only valid in the turbulent boundary layer.

According to the conditions of excitation process [equation (16) or (17)], $A+B>0$ when with P0 sufficiently close to zero ($P_0 = \vec{\zeta}_a \cdot \nabla\theta^* \cong 0$), (A+B) can excite a weather system. In mid-latitude large-meso scale movements, $\overrightarrow{\varsigma_a} \cong$ a constant vector and is almost parallel to the vertical coordinate. Therefore, the above conditions can be simplified as $A+B>0$ with $\partial\theta^*/\partial z$ sufficiently close to zero ($A+B>0$ with $\partial\theta^*/\partial z \cong 0$). From the expression of θ^*, A and B, (4), (8) and (9), the conditions of dry excitation can be simplified as ($\nabla Q > 0$ with $\partial\theta/\partial z \cong 0$); the conditions of moist excitation is that the θ_m^* gradient vector has a component counter to the direction of the baroclinic vector with $\partial\theta_m^*/\partial z \cong 0$ ($(\nabla p \times \nabla\theta)\cdot\nabla\theta^* > 0$with $\partial\theta_m^*/\partial z \cong 0$) and the conditions of ice excitation is ($(\nabla p \times \nabla\theta)\cdot\nabla\theta^* > 0$ with $\partial\theta_i^*/\partial z \cong 0$). Here $\theta_m^* = \theta\exp\left(\frac{L_d}{C_p T} r^k q_s\right)$ and $\theta_i^* = \theta\exp\left(\frac{(L_i + L_d)}{C_p T} r^k q_s\right)$ are moist and ice generalized potential temperature respectively. When r=1, $\theta_m^* = \theta\exp\left(\frac{L_d q_s}{C_p T}\right) = \theta_{se}$ and $\theta_i^* = \theta\exp\left(\frac{(L_i + L_d)}{C_p T} q_s\right) = \theta_{ise}$.

Under the right conditions [($\nabla Q > 0$ with $\partial\theta/\partial z \cong 0$ from the Earth's surface up) or the forcing of the Earth's surface], a dry convective positive vorticity system occurs. Above its top, $\partial\theta/\partial z \cong 0$ continues to quickly transform into and the dry convective positive vorticity system or the dry GPV non conserved process continues to stretch upward given that with is satisfied. Once the dry convective positive vorticity system reaches the condensation level and the rising air parcel encouraged by the system enters the subsaturation state, the cumulus occurs.

In the cumulus (Cumulus humilis and Cumulus congestus), when the rising air parcel reaches the condensation level and enters the subsaturation state, if the sufficient and necessary condition of the moist excitation process [the air in the subsaturated state, $(\nabla p \times \nabla\theta)\cdot\nabla\theta^* > 0$ and $\partial\theta_m^*/\partial z \cong 0$] is satisfied, the air in the subsaturated state quickly becomes saturated and $\partial\theta_m^*/\partial z \cong 0$ quickly transforms into $\partial\theta_m^*/\partial z = 0$, so both the condensation process and a moist convective positive vorticity system (cumulus cloud) occur.

According to the scale transformation mechanism, this system exhibits an increase in GPV from P_d (GPV in the dry area) to P_s (GPV in the saturated area) coupled with rising air moving from dry areas into saturated areas through a very thin sub-saturated area at the cloud base (the green line in Figure 1). Contrarily, a decrease in GPV from P_s to P_d may be observed, coupled with rising air moving from saturated areas into dry areas through a very thin sub-saturated area at the cloud top (the green line in Figure 1a). However, in the cloud, GPV is conservative because $\theta^* = \theta_{se}$ and $\partial\theta_m^*/\partial z = \partial\theta_{se}/\partial z = 0$ (Figure 1a). Above the cloud top, the rising air in the sub-saturated state continues to quickly transform into the saturated state and $\partial\theta_m^*/\partial z \cong 0$ continues to quickly transform into $\partial\theta_{se}/\partial z = 0$, so cumulus cloud or the moist GPV non conserved process continues to stretch and develop upward (the GPV conservative saturated area becomes higher and larger coupled with the expansion of a thin GPV non-conservative sub-saturated area), given that the condition (the air in the sub-saturated state, $(\nabla p \times \nabla\theta)\cdot\nabla\theta^* > 0$ and $\partial\theta_m^*/\partial z \cong 0$) is satisfied.

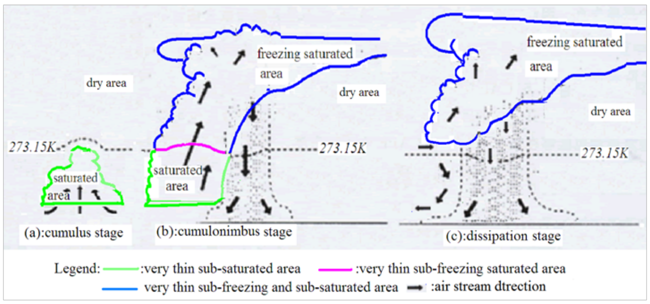

Figure 1: A schematic diagram of the dynamic analysis of convection stages.

Note: The modified figure according to the theoretical analysis of D18-0213 in the drawings provided by Doswell and Emanuel [28,29].

Once the ascending height of cumulus cloud results in a temperature drops below 277.13K, according to Wang Huang [14], the satisfaction of the random temperature variable $T_x \leq 273.15K$ and the condensation condition directly transforms the water vapor into solid water (deposition). That is, the freezing phenomenon with the freezing probability φ (T) appears, which begins the ice excitation process, it is, cumulonimbus occurs.

In the cumulonimbus (Cumulonimbus calvus and Cumulonimbus capillatus), as the ascending height of cumulus cloud causes the temperature T to drop below 277.13K, the ice excitation process proceeds if the sufficient and necessary condition of the ice excitation process (the air in sub-freezing saturated state or the subfreezing and sub-saturated state, $(\nabla p \times \nabla \theta) \cdot \nabla \theta^* > 0$ with $\partial \theta_i^* / \partial z \cong 0$) is still satisfied above the cloud top. The air in sub-freezing saturated state or the sub-freezing and sub-saturated state is quickly becomes the freezing saturated state and $\partial \theta_i^* / \partial z \cong 0$ quickly transforms into $\partial \theta_i^* / \partial z = \partial \theta_{ise}{}^* / \partial z = 0$, so both the freezing process (the water vapor with the airborne liquid water in cloud satisfying $T_x \le 273.15K$ is directly transformed into solid water) and a ice convective positive vorticity system (cumulonimbus cloud) occur. In this system, an increase in GPV from P_d to P_s is observed coupled with rising air movements from dry areas into saturated areas through a very thin sub-saturated area at the cloud base, which is similar to the moist convective positive vorticity system in the cumulus. However, above the saturated area, there is an additional increase in GPV from P_s to P_{sf} [GPV in the freezing saturated area, (r = 1 and T < 269.13K)] coupled with rising air movement from the saturated area into the freezing saturated area through a very thin sub-freezing saturated area ($277.13K > T > 269.13K$ and r = 1) near the 273.15K isothermal surface (red line in Figure 1b). There is also a correspondingly larger decrease in GPV from P_{sf} to P_s (not from P_s to P_d) coupled with rising air movement from the freezing saturated to dry area through very thin sub-freezing and sub-saturated area (blue line in Figure 1) at the cloud top (Figure 1b). However, in the freezing saturated area, the GPV is once again conservative because $\theta^* = \theta \exp\left((L_i + L_d) q_s / C_p T\right) = \theta_{ise}$. If the sufficient and necessary condition of the ice excitation process is satisfied, the rising air in the subfreezing and subsaturated states above the cloud top continues to quickly move into the freezing saturated state. This system or cloud continues to develop upward, Cumulus congestus becomes Cumulonimbus calvus and then becomes Cumulonimbus capillatus.

At this stage, cumulus surrounding the earliest cumulonimbus will quickly merge via the convergence air stream, because

1. The additional increase of earliest cumulonimbus in GPV (from to through the very thin sub-freezing saturated area) makes its upward vertical velocity more powerful than any other cumulus
2. Unlike the very thin sub-saturated area, the very thin sub-freezing saturated area is close to the 277.13K isothermal surface and extends from the inner to outer regions almost horizontally. This creates an increasingly extensive early cumulonimbus.

From Wang et al, (2006) and Cheng et al, (2003)[24,25], if the upward cloud top of cumulus or cumulonimbus meets the sinking cold moist air stream from the CIN top, the rising air compels the sinking cold moist air stream to stop sinking and to rise such that the height satisfying the sufficient and necessary condition of the moist or ice excitation process can quickly extend to the original height of the sinking cold moist air stream above the CIN.

When the sufficient and necessary condition of the moist or ice excitation process is no longer satisfied, the moist or ice convective positive vorticity system returns to a new evolution process at the meso-microscale (not at large scales) and is in a thermodynamically steady state for an extended period of time. In the most vigorous stage of Cumulonimbus capillatus, the cloud top shape often becomes anvil-shaped and horizontally spread in the direction of the wind because of the inhibition of a strong stable layer above it and the impact of its upper maximum wind speed.

In addition, when cumulus or cumulonimbus is in a thermodynamically steady state, $A + B \cong 0$ with $\partial \theta^* / \partial z \cong 0$ in very thin sub-saturated or sub-freezing and sub-saturated area of cloud but $A + B \cong 0$ with $\partial \theta^* / \partial z = 0$ in cloud. If the θ^* gradient having a component in (no longer counter to) the direction of the baroclinic vector, it is, $(\nabla p \times \nabla \theta) \cdot \nabla \theta^* < 0$ (A<0) at the sub-saturated or subfreezing and sub-saturated area of cloud base, from the sufficient and necessary condition of elimination process the convergence sinking airflow (caused by the decrease (instead of increase) in GPV and the occurrence of convective inverse vorticity system) will instead be observed upward from the cloud base. Once so, the convective cloud dissipation stage begins.

$$\begin{cases} O\left(\dfrac{f^{-1}(A+B)}{\partial \theta^* / \partial z}\right) > \max\left(O(R), O\left(\dfrac{W}{fH}\right)\right) \\ (A+B) < 0 \end{cases} \quad (23),$$

In the convective cloud dissipation stage, the original rising air at the cloud base will no longer rise and instead sink because of convective inverse vorticity system resulting convergence sinking airflow. Almost all the airborne liquid water drops or solid ice crystals will then fall in the form of precipitation [30] and the air in the sub-saturated state or sub-freezing and sub-saturated state will quickly moves into the dry air state. Likewise, the air in the saturated state or freezing saturated state quickly moves into the sub-saturated state or sub-freezing and sub-saturated state, and the cloud base quickly moves upward (Figure 1c). The dissipation process of cumulus or cumulonimbus continues until the sufficient and necessary condition of the moist or ice elimination process is no longer satisfied. Even so, another kind of dissipation process for cumulus or cumulonimbus, it is, the adaptation process, continues until cumulus or cumulonimbus has completely disappeared. As lost is the condensation or freezing processes of water vapor caused by the convergence of the rising airflow below the cloud, cumulus or cumulonimbus no longer presents an open system. Rather, it exhibits a quasi-closed system. In this system, the thermodynamically steady state is broken and the excitation impact caused by A+B can be omitted. The sufficient and necessary condition of the adaptation process is satisfied, and so the non-equilibrium of the air pressure gradient force, Coriolis force, inertial force, and gravity disappears by rapid adaptation process. This adaptation process and the frictional effect results in the rapid and complete disappearance of the original meso-microscale systems related to cumulus or cumulonimbus to again generate large scale air movement.

Conclusion

To figure out the influence of non-conserved GPV caused by latent heating and dry diabatic heating on convective weather, by comparing latent heating and dry diabatic heating term (A+B) with nonlinear advection term, three kinds of processes are disscussed, namely adaptation, evolution, and excitation (elimination), associated with the conserved, quasi-conserved, and non-conserved GPV, respectively.

For each transformation process of convective weather, its dynamic causes, conditions and main features are theoretically analyzed and described, many puzzles about convective may be understood and explained.

A particularly important finding in the analysis suggests that cumulus surrounding the earliest cumulonimbus quickly merges via a convergent air stream during the cumulonimbus stage, which makes the cumulonimbus increasingly extensive.

The present study only offers preliminary work. In order to break down the restrictions of the conservation law of the potential vortices to make greater progress in convective weather forecasting and understanding, a lot of space and effort is still needed to perfect this huge project.

First, establish the numerical simulation models based on generalized dynamic equations related to condensation & freezing processes to confirm the dynamic causes, conditions and main features of each convective cloud pointed out in this paper.

Secondly, according to theoretical work of dynamic analysis of convective weather in this paper, design and produce charts showing the thickness of CIN and the trace process of CIN in the vertical and horizontal planes to determine the intensity, time and location of the strong convective weather that will occur.

Thirdly, according to theoretical work of dynamic analysis of convective weather in this paper, revisit convective scheme or even parameterization to improve them.

Therefore, as the beginning of the entire huge research process, due to the limitation of the thesis, our current work can only be the mathematical reasoning on one idealized figure based on the first principles.

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