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Brief Report

A Framework for Solving the Optimal Screening Interval for Tuberculosis

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Abstract

Tuberculosis is one of the major threats to human health. Regular screening is helpful for early detection of the infection, which can make treatment easier and also prevent from unintended infection to others. A model is constructed to depict the infection of tuberculosis and solve the optimal screening interval which minimizes a cost function. Illustrative examples are presented.

Keywords: Infective diseases; Screening interval; Treatment; Tuberculosis

Introduction

People can get infected with tuberculosis if they frequently meet with tuberculosis carriers who happen to be in bacilliferous state, especially in places with bad air circulation [1]. The vulnerability depends on the efficiency of one's immune system, the frequency it meets bacilliferous tuberculosis carriers, the duration they spend together, the closeness of them and the air circulation. Regular screening is useful for early detection of tuberculosis, so that the treatment will be easier and it can prevent an infected person to infect others unintentionally [2]. Say, a person who knows that it has tuberculosis can choose to wear a mask when it meets people so that others are not infected. The screening methods for tuberculosis include computed tomography and sputum smear. The determination of an appropriate screening interval is important because too frequent screening may lead to a waste of medical resources, whereas insufficient frequency may make more people to be infected and the tuberculosis may also increase the risk of lung cancer [3]. The optimal inspection interval has been studied widely for early detection diagnosis of defects in engineering systems, but relatively rare for screening of diseases [4-7].

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In this paper, we construct a model to depict the infection of tuberculosis and calculate the total cost associated with a specific screening interval. The total costs include the following three components: 1) the cost of screening, 2) the cost associated with treatment if an individual is infected and 3) the cost associated with onward transmission between the time of infection and the time of treatment. Optimal screening which minimizes the cost function is solved for illustrative examples.

The remainder of the paper is written as follows. Section 2 outlines the assumptions and notations used in our models. Section 3 features the main description of the models. Numerical examples are given in section 4. Finally, section 5 concludes the paper.

Notations and Assumptions

Notations

We define the following notations for the models described in our study. The unit of time is years.

L	The random variable for the exposure period
$h(L)$	The probability density function of the exposure period
$H(L)$	The cumulative density function of the exposure period
M	The upper limit of the exposure period
T	The periodic screening interval
n	The maximum number of screenings per person
λ	The rate of meeting people
N	Expected number of people in a crowded area
r	The proportion of infected people in the population
p	The probability of infection
C_s	The cost of each screening
C_c	The treatment cost as a function as the infection time and delay time between infection to detection
C_o	The cost associated with onward transmission
X	The random variable for the time of infection
f	The probability density function of the time of infection
$F(X)$	The cumulative density function of the time of infection

Assumptions

The following assumptions are proposed for the cost model.

- The exposure period L follows a distribution in a given interval $[0,M]$

- A person meets others with rate λ per unit time. The number of people it meets is poisson distributed with parameter N. The infection probability p is an increasing function of the number of infected people in the area

- The proportion of infected people in the population is r. For simplicity, it is assumed that all and only the infected people are in bacilliferous state

- A screening occurs every T time units, costs C_s units and requires negligible time

- A person can only be infected at most once throughout the exposure period

- Once a person is identified as being infected, screening stops for him (or her) and treatment is initiated. Also, the person will not infect others, as it either not meet others or it meets others with a mask

Models

Based on assumptions, it can be concluded that the infection rate for a healthy person is

$$\sum_{k=1}^{\infty} \frac{e^{-N} N^k}{} \sum_{i=1}^{} \binom{}{} r^i (1-r)^{k-i} p(i).$$

If X is the time of infection, the probability density function of X is

$$f(x) = \begin{cases} \lambda \sum_{k=1}^{\infty} \frac{e^{-N} N^k}{k!} \sum_{i=1}^{k} \binom{k}{i} r^i (1-r)^{k-i} p(i) e^{-\lambda \sum_{k=1}^{\infty} \frac{e^{-N} N^k}{k!} \sum_{i=1}^{k} \binom{k}{i} r^i (1-r)^{k-i} p(i) x} & x > 0, \\ 0 & x \le 0. \end{cases} \quad (1)$$

The length of the exposure period L has a truncated distribution derived from life tables established by the China life insurance industry. This distribution is an approximation of the real age distribution of the population from birth to a given age. It can be converted to other forms to match different populations.

$$h(l) = \frac{0.006 e^{0.3l}}{\int_0^M 0.006 e^{0.3l}} \qquad l \in [0, \mathrm{M}]. \qquad (2)$$

As shown in figure 1, we consider three scenarios in this model and n=[l/T] is a variable representing the maximum number of screenings per person. The expected cost of every scenario is specified as follows.
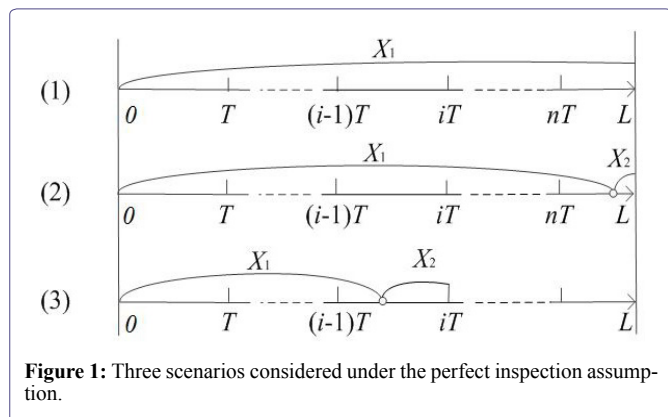

Figure 1: Three scenarios considered under the perfect inspection assumption.

1. Scenario (1): Never infected throughout the exposure period

As shown in figure 1, there is a probability $P(X>L)$ that an individual never gets infected throughout the exposure period. The only cost for such an individual is the cost of screening, which is written as

$$E(C_1^1(T)) = \int_0^M h(l)(1-F(l)) n C_s \, dl \qquad (3)$$

2. Scenario (2): Infected after the last screening

If a person is infected at some time later than the last screening received, then the infection cannot be detected. Therefore, the costs incurred include the cost of screening and the cost of onward transmission, but no treatment cost. The probability of this scenario is $P(nT<X<L)$. Hence, the expected cost is

$$E(C_1^2(T)) = \int_0^M h(l) \int_{nT}^l f(x)(n C_S + (l-x)\lambda C_o \sum_{k=1}^{\infty} \frac{e^{-N} N^k}{k!} \sum_{i=1}^{k} \binom{k}{i} \cdot$$

$$r^i (1-r)^{k-i}(k-i)(p(i+1) - p(i))) dx dl \qquad (4)$$

Where, C_o is the burden to society if one more person is infected with tuberculosis.

3. Scenario (3): Infected before the last screening

Based on the assumption of perfect inspection, if an infection occurs before the last screening, it will be detected at the first screening following infection. The number of screenings is $[x/T]+1$ since screening stops after treatment. The costs in this scenario include the cost of screening, the cost of treatment and the cost of onward transmission from the time of infection to the time of detection. The expected cost is

$$E(C_1^3[T]) = \int_T^M h(l) \int_0^{nT} f(x)(([x/T]+1)C_S + C_c(x, ([x/T]+1)T - x)$$

$$+ (([x/T]+1)T - x)\lambda C_o \sum_{k=1}^{\infty} \frac{e^{-N} N^k}{k!} \sum_{i=1}^{k} \binom{k}{i} r^i (1-r)^{k-i}(k-i)(p(i+1) - p(i))) dx dl$$

Overall, we combine the costs of the three scenarios to get the expected total cost function:

$$E(C_1(T)) = E(C_1^1(T)) + E(C_1^2(T)) + E(C_1^3(T)). \qquad (6)$$

The optimal screening interval can be determined by minimizing the total cost.

Conclusion

In this paper, a framework is proposed to solve the optimal screening interval for tuberculosis. The total cost associated with a specific screening interval include screening cost, treatment cost and the burden to the society if one gets infected and further infect others. In the future, the forms of the functions involved in this model need to be figured out and their parameters need to be estimated. In addition, it is interesting to consider not only tuberculosis, but all the major types of infectious and non-infectious diseases to solve the optimal interval for body checking.

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